

**Exam ONE, MTH 205, Summer 2009**

Ayman Badawi

**QUESTION 1. (Each = 6 points, total 36 points)**

(i) Find  $\ell\{U(x - 3)e^{2x}\}$

(ii) Find  $\ell\left\{\int_0^x e^{-3r} \sin(2r) dr\right\}$

(iii) Find  $\ell^{-1}\left\{\frac{s+10}{(s+4)^4}\right\}$

(iv) Find  $\ell^{-1} \left\{ \frac{e^{-s}}{(s-1)^2+1} \right\}$

(v) Use CONVOLUTION to find  $\ell^{-1} \left\{ \frac{2s}{(s^2+4)^2} \right\}$  ( Hint: you may need  $\sin(a)\cos(b) = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$ )

(vi) Find  $\ell^{-1} \left\{ \frac{8e^{-3s}}{s^2-4} \right\}$

**QUESTION 2. (18 points)** Let

$$k(x) = \begin{cases} 0 & 0 \leq x < 4 \\ 6 & x \geq 4 \end{cases}$$

Solve:  $y^{(2)} - 3y' + 2y = k(x)$ ,  $y(0) = 0$  and  $y'(0) = 0$ .

**QUESTION 3. (20 points)** Solve:  $y'(x) = xe^x + \int_0^x 2e^{x-r}y(r) dr$ ,  $y(0) = 0$

**QUESTION 4. (18 points)** solve for  $x(t)$  and  $y(t)$  (use any method ) such that

$$x'(t) - y(t) = 2t$$

$$x(t) - \int_0^t y(r) dr = t^2, x(0) = 0.$$

**QUESTION 5. (8 points)** Find the largest interval around  $x = 2$  such that  $(\sqrt{6-x})y^{(2)} + \frac{3}{x-1}y' + y = \frac{5}{x-7}, y(2) = 0, y'(2) = -1$  has a unique solution.

### Faculty information

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